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SPIN STRUCTURES ON HYPERSURFACES AND THE SPECTRUM OF THE DIRAC OPERATOR ON SPHERES*

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Abstract. Recent results on pin structures on hypersurfaces in spin manifolds are reviewed. A new form of the Dirac operator is used to compute its spectrum on n-dimensional spheres. This constrbution is based on two papers by the author, where details and proofs can be found (Ref.4 and 5).

1. This research has been motivated by, and can be summarized in, the following observations:
(i) In odd dimensions, it is appropriate to use the *twisted* adjoint representation $\rho : \text{Pin}(n) \to \text{O}(n)$ to find a cover of the full orthogonal group $\text{O}(n)$ which extends the standard homomorphism $\text{Spin}(n) \to \text{SO}(n)$. Here ρ is given by $\rho(a)v = \alpha(a)va^{-1}$, where $v \in \mathbf{R}^n$, $a \in \text{Pin}(n) \subset \text{Cl}(n)$ and α is the grading (main) automorphism of the Clifford algebra $\text{Cl}(n)$ [1]. Using the twisted representation leads to modifying the Dirac operator [2].
(ii) The bundles of "Dirac spinors" over even-dimensional spheres are trivial [3]; this observation generalizes to hypersurfaces in $\mathbf{R}^{n+1}$: every such hypersurface, even if it is non-orientable, admits a pin structure with a trivial bundle of Dirac (n even) or Pauli (n odd) spinors [4]
(iii) The spectrum and the eigenfunctions of the Laplace operator Δ on the n-dimensional unit sphere $\mathbf{S}_n$ are easily obtained from the formula

$$\sum_{i=1}^{n+1} \partial^2/\partial x_i^2 = r^{-2}\Delta + r^{-n}\partial/\partial r(r^n\partial/\partial r) \tag{1}$$

This formula generalizes to a foliation of $\mathbf{R}^{n+1}$ by hypersurfaces and extends to the Dirac operator, allowing a simple computation of the Dirac spectrum of n-spheres [5].

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