GENERALITIES ON GEOMETRIC THEORIES OF GRAVITATION

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All classical, local theories of spacetime and gravitation are based on a rather small number of assumptions about the geometry, the form of the field equations and the nature of the sources. The basic assumptions may be formulated in such a way as to allow an easy comparison between the theories. To achieve this, it is convenient to distinguish the 'kinematic' part of the assumptions, referring to the type of geometry, from the 'dynamic' part, which consists in specifying the form of the field equations.

The kinematics of essentially all theories is based on a four-dimensional differentiable manifold M as the model of spacetime. The manifold is endowed with at least two geometric structures: a connection and a metric structure. The connection is necessary to compare - and, in particular, to differentiate - objects such as vectors and tensors, needed to describe momenta, forces, field strengths, etc. In most cases a linear connection is used, but it is possible to develop all or parts of physics on the basis of other connections (affine, conformal). For example, to compare directions and to define straight (autoparallel) lines it suffices to consider a projective connection, defined as the equivalence class of linear connections whose coefficients $\Gamma^{\mu}_{\nu\rho}$ are related by

$$\bar{\Gamma}^{\mu}_{\nu\rho} = \Gamma^{\mu}_{\nu\rho} + \delta^{\mu}_{\nu}\lambda_{\rho} \quad .$$

A metric structure is needed to measure distances, time intervals, angles and relative velocities. A theory is relativistic if its metric structure is given by a metric tensor g of signature (+++-). A somewhat weaker metric structure is called conformal geometry: it is given by an equivalence class of metric