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THEORETICAL PHYSICS

On the Einstein—Cartan Equations. II.

by

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Summary. It is shown that by covariant exterior differentiation of the equations of the generalized theory of gravity one arrives at a system of relations which hold as an algebraic consequence of the field equations themselves. The Dirac equation in a Riemann-Cartan space-time is written in a manner adapted to the calculus of exterior forms.

1. The differential identities. The curvature and torsion forms of any affinely connected space satisfy the Bianchi identities*)

$$D\Omega^k_l = 0 \quad \text{and} \quad D\Theta^k = \Omega^k_l \wedge \theta^l.$$

These identities may be used to evaluate the covariant exterior derivatives of the 3-forms e_j and c_{kl} appearing in the Einstein—Cartan equations of the generalized, metric theory of gravitation. It follows directly from (I.11) and (I.12) that

$$De_j = \tfrac{1}{2}\eta_{jklm}\,\Theta^m \wedge \Omega^{kl}, \tag{1}$$

$$Dc_{kl} = \eta_{lj} \wedge \Omega^j_k - \eta_{kj} \wedge \Omega^j_l. \tag{2}$$

Eq. (1) is the generalization, to the Riemann-Cartan space, of the 'contracted Bianchi identity' $De_j=0$ which plays an important role in Einstein's theory of gravitation. The right-hand sides of Eqs (1) and (2) may be rearranged to give the formulae

$$De_j = Q^k_j \wedge e_k + \tfrac{1}{2} R^{kl}{}_j \wedge c_{kl}, \tag{3}$$

$$Dc_{kl} = \theta_k \wedge e_l - \theta_l \wedge e_k, \tag{4}$$

where

$$\tfrac{1}{2}\theta^j \wedge Q^k_j = \Theta^k \quad \text{and} \quad \tfrac{1}{2}\theta^j \wedge R^{kl}{}_j = \Omega^{kl}.$$

Let σ be a representation of the Lorentz group in $\mathbf{R}^N$ and let (σ^{Bl}_{Ak}) be the matrix corresponding to the derived homomorphism of Lie algebras. The matrix $\sigma_{kl} =$

*) In this note, similar assumptions are made, and the same notation is used, as in Part I [1]. Equations appearing there are referred to in the style (I.n). The frames (θ^k) are assumed to be orthonormal so that $g_{kl}=0$ for $k \neq l$ and $g_{11}=g_{22}=g_{33}=-g_{44}=-1$.