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THEORETICAL PHYSICS

On the Einstein—Cartan Equations. I*)

by

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Summary. A new argument is presented to support the Cartan idea of modifying Einstein's theory of gravitation by relating torsion to intrinsic angular momentum. It is shown that the Einstein equation may be written in two equivalent forms, with either the symmetric or the canonical tensor density of energy and momentum as the source. The Cartan equation determines the linear connection only up to projective transformations; this arbitrariness may be removed by requiring that the connection be metric.

1. Introduction. In 1922 Élie Cartan [1] suggested a simple generalization of Einstein's theory of gravitation. He proposed to consider, as a model of space-time, a four-dimensional differential manifold with a metric tensor and a linear connection compatible with the metric but not symmetric, in general. According to Cartan, the torsion tensor of the connection should be related to the density of intrinsic angular momentum [2]. Independently of Cartan, similar ideas were put forward by several authors (for example, see [3—5]; the last paper contains other relevant references).

The following is a heuristic argument to support the Cartan proposal: by the holonomy theorems, curvature and torsion are related, respectively, to the groups of homogeneous transformations and of translations in the tangent spaces of a manifold. In the approximation of special relativity, the group of inhomogeneous Lorentz transformations and its invariants (mass and spin) play a fundamental role in the description of elementary physical phenomena. In Einstein's theory of general relativity, mass directly influences curvature but spin has no similar dynamical effect. As a result of the absence of torsion, the infinitesimal holonomy groups of the Cartan connection of an Einstein space consist of only homogeneous transformations [6]. By introducing torsion and relating it to spin, one obtains an interesting link between the theory of gravitation and the theory of special relativity [7].

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