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GOLDEN OLDIE EDITORIAL

Editorial note to: J. L. Synge, On the deviation of geodesics and null geodesics, particularly in relation to the properties of spaces of constant curvature and indefinite line-element

and to: F. A. E. Pirani, On the physical significance of the Riemann tensor

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1. The papers by J. L. Synge and F. A. E. Pirani reprinted here played a major role in the understanding of the subtle relations between the geometrical and physical aspects of General Relativity Theory. Both papers make use of the equation of geodetic deviation. Synge's paper [1], published in 1934 in the prestigious *Annals of Mathematics*, is the first publication where deviation of *null* geodesics is considered. A new result presented there consists in showing how the geodetic deviation equation can be used to obtain a new characterization of the sectional curvature in a (pseudo) Riemannian space (M, g). The equation is solved in spaces of constant curvature with g of Lorentzian signature $(+ - --)$ and used to analyze the global properties of these de Sitter and anti-de Sitter spaces. The exposition in Synge's paper is so concise and lucid that it does not require any summary or additional comments. Most of its results have been incorporated into the book [2]. Global properties of the de Sitter space are thoroughly discussed in [3].

The republications of the original Synge and Pirani papers can be found in this issue following the editorial note and online via doi:10.1007/s10714-009-0786-x (Synge) and via doi:10.1007/s10714-009-0787-9 (Pirani).

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The paper by Pirani, written during his stay with Synge in Dublin, emphasizes the usefulness of tetrads in relativity and explains how the Riemann tensor can be measured by reference to the deviation of geodesics and with the help of a simple model of a spinning body.

2. The *equation of geodetic deviation* is a simple and convenient tool to study deformations of geodesics, the occurrence of conjugate points on geodesics and the global properties of (pseudo) Riemannian manifolds. In differential geometry, geodesics are defined either as autoparallels or as lines of extremal length. The first definition requires only a linear connection; the second is based on a metric tensor. In Riemannian geometry, the two definitions coincide in the sense that autoparallels of the Levi-Civita connection are extremals of the length or energy integral. The equation of geodetic deviation in n-dimensional Riemannian geometry was first derived by Levi-Civita [4] and Synge [5,6].

Let M be an n-dimensional manifold with a symmetric linear connection; the corresponding covariant derivative in the direction of the vector field X is denoted by ∇_X. The equation of deviation of autoparallels easily follows from the definition of the curvature tensor R in the form given, e.g., in Chapter 11 of [7],

$$(\nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]})Z = R(X, Y)Z \tag{1}$$

for all vector fields X, Y and Z on M. Indeed, let the vector field X be such that its trajectories are affinely parametrized autoparallels, so that $\nabla_X X = 0$. If Y is a vector field commuting with X, then, since torsion is assumed to vanish,

$$\nabla_X Y = \nabla_Y X, \tag{2}$$

one has $\nabla_X^2 Y = \nabla_X \nabla_Y X$ and Eq. (1) for $Z = X$ gives the equation of deviation of autoparallels

$$\nabla_X^2 Y - R(X, Y)X = 0. \tag{3}$$

In this form, the deviation equation is applicable in affinely connected, but not necessarily Riemannian, manifolds such as, for example, the space–time of Newtonian gravitation [8,9]. In this case, Eq. (3) leads to the interpretation of curvature as the tide-producing gravitational 'force'; see Chapter 12 in [7].

In a (pseudo) Riemannian space (M, g), the covariant derivative is defined by the metric and symmetric Levi-Civita connection. By a choice of the affine parameter along the trajectories of X, one can achieve $g(X, X) = c$ on M, where $c \in \{1, -1, 0\}$. Differentiating both sides of the last equation in the direction of Y and using (2), one obtains $\nabla_X(g(X, Y)) = 0$ for every vector field Y commuting with the geodetic vector field X: the scalar product $g(X, Y)$ is constant along the trajectories of X. Moreover, if the vector field X is not null, then the vector field $Y' = Y - g(X, Y)X/c$ also commutes with X and is everywhere orthogonal to X.

3. The deviation equation for geodesics in Riemannian geometry is often called the *Jacobi equation* even though Carl Gustav Jacob Jacobi died 3 years before Georg Friedrich Bernhard Riemann delivered his famous habilitation lecture that started the

new field of geometry. The Jacobi equation is used to define the important notion of conjugate points on geodesics. Since it is rather different from the derivation of (3), it is worth recalling Jacobi's [10] original line of thought in the simplest case of the action integral,

$$I(x) = \int_{t_1}^{t_2} L(t, x(t), \dot{x}(t))\mathrm{d}t, \quad \dot{x} = \mathrm{d}x/\mathrm{d}t, \tag{4}$$

under the assumption of non-degeneracy, $\partial^2 L/\partial \dot{x}^2 \neq 0$. The corresponding Euler–Lagrange equation defines the extremals of (4). The second variation of (4),

$$J(x, y) = \frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}s^2} I(x + sy)|_{s=0},$$

induced by a function y, is

$$J(x, y) = \frac{1}{2}\int_{t_1}^{t_2} \left(\frac{\partial^2 L}{\partial x^2} y^2 + 2\frac{\partial^2 L}{\partial x \partial \dot{x}} y\dot{y} + \frac{\partial^2 L}{\partial \dot{x}^2}\dot{y}^2\right)\mathrm{d}t. \tag{5}$$

In order that an extremal x of (4) be a local minimum of the action integral, the inequality $J(x, y) \geq 0$ must hold for every non-zero function y vanishing at t_1 and t_2. Moreover, since one can choose y so that y^2 is 'small' and $\dot{y}^2$ is 'large', a necessary condition for x to be a minimum is the *Legendre condition*

$$\frac{\partial^2 L}{\partial \dot{x}^2}(t, x(t), \dot{x}(t)) \geq 0 \quad \text{for } t_1 \leq t \leq t_2.$$

The function x being now fixed, one considers $J(x, Y)$ as an action integral for the function Y; the corresponding Euler–Lagrange equation is the *Jacobi equation* associated with (4). It is a linear equation for the function Y,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial^2 L}{\partial \dot{x}^2}\dot{Y}\right) + \left(\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial^2 L}{\partial x \partial \dot{x}} - \frac{\partial^2 L}{\partial x^2}\right) Y = 0. \tag{6}$$

Given an extremal x, a point $x(t)$ on it, $t_1 < t \leq t_2$, is said to be *conjugate* to $x(t_1)$ if there is a non-zero solution Y of (6) such that $Y(t_1) = Y(t) = 0$. If there are no conjugate points in the interval (t_1, t_2), then Eq. (6) has a solution Y nowhere zero for $t_0 \leq t \leq t_1$ and the function

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(y^2\left(\frac{\partial^2 L}{\partial x \partial \dot{x}} + \frac{\partial^2 L}{\partial \dot{x}^2}\frac{\dot{Y}}{Y}\right)\right)$$

can be subtracted from the integrand of (5) without altering the value of the integral (5) which then becomes

$$J(x, y) = \frac{1}{2}\int_{t_1}^{t_2} \frac{1}{Y^2}\frac{\partial^2 L}{\partial \dot{x}^2}(\dot{y}Y - y\dot{Y})^2 \mathrm{d}t.$$

By combining this observation with a strengthened form of the Lagrange condition, $\partial^2 L/\partial \dot{x}^2 > 0$, one obtains a sufficient *Jacobi condition* for the existence of a weak minimum of (4); see §23 in [11]. An easy generalization is to consider n dependent variables x^μ, $\mu = 1, \ldots, n$. If L is the 'energy' function $g_{\mu\nu}(x)\dot{x}^\mu \dot{x}^\nu$, or its square root (the length function), the Euler–Lagrange equation is that of geodetic lines,

$$g_{\mu\nu}\frac{D}{\mathrm{d}t}\dot{x}^\nu = 0, \quad \frac{D}{\mathrm{d}t} = \nabla_{\dot{x}},$$

and the Jacobi equation (6) becomes the restriction of (3) to a single geodesic, with $X^\mu = \dot{x}^\mu$ and $Y = (Y^\mu)$,

$$\frac{D^2}{\mathrm{d}t^2}Y^\mu - R^\mu{}_{\nu\rho\sigma}X^\nu X^\rho Y^\sigma = 0. \tag{7}$$

A solution Y of (7) is called a *Jacobi field* along the geodesic x. The flow of a vector field Y commuting with X permutes the geodesics (trajectories) of X. Therefore, if there is a Jacobi field commuting with X and vanishing at two points on one geodesic x, then there is a one-parameter family of geodesics through these two conjugate points.

In relativity, with g of Lorentzian signature, time-like geodesics locally provide a maximum of the action integral and the Jacobi condition has to be appropriately modified; this is done, with full proofs, in §4.5 of [12].

The geodetic deviation equation and the analysis of conjugate points on geodesics found most important applications in general relativity and cosmology in the study of singularities that appear in the process of gravitational collapse and the formation of black holes. The fundamental theorems on singularities by Penrose [13], Hawking [14,15], and Geroch [16] are now so well presented in books [12,17,18] that it would not be appropriate to summarize them here.

A simple, but physically significant, application of the deviation equation is to study the behavior of a cloud of particles under the influence of a 'sandwich' plane gravitational wave; see §9.1 in [19].

4. Felix Pirani starts [20] by making a case for referring observations to *orthonormal tetrads* rather than to local, curvilinear coordinates. This point of view has since been accepted by the relativity community, as can be seen by its exposition in textbooks; see, e.g., Chapters 6 and 8 in [7]. An important extension of the use of tetrads occurred at the beginning of the 1960s when Sachs [21], Newman and Penrose [22] introduced *null* tetrads and used them as a basis for a calculus of 'spin coefficients' that has become the standard tool in the study of gravitational fields. In Sect. 2, the

author explains the physical significance of the *Fermi–Walker transport* [23,24] of tetrads associated with the motion of an observer that is not necessarily in free fall. He shows that this transport is the relativistic analog of the method of transporting space axes so that they have fixed directions in absolute space of the Newtonian theory. Pirani points out that, by observing a (sufficiently large) number of particles and using the deviation equation (7), an observer can determine the full Riemann tensor in the vicinity of his world-line. This is true, but not so obvious because (7) contains the components of the Riemann tensor in the combination $R^{\mu}{}_{\nu\rho\sigma} + R^{\mu}{}_{\rho\nu\sigma}$; see Exercise 11.7 in [7]. In Sect. 3, the general considerations are illustrated by an analysis of the Fermi–Walker transport in the Schwarzschild space–time. A vector so transported along a circular path undergoes a secular rotation that is a combination of the special-relativistic Thomas precession and of an 'inertial drag' due to the mass of the central body. These considerations are extended, in Sect. 4, to the motion of a spinning test particle. Pirani attributes the derivation of its equations to Achilles Papapetrou (1951). However, the equation of motion of classical spin (Eq. (4.2) in Pirani's paper) was derived, in the context of special relativity, by Frenkel [25] who referred to Thomas [26] as its originator. The full set (4.1) and (4.2) was derived, in general relativity, by Mathisson [27]. Both Frenkel and Mathisson used Eq. (4.3) as the additional condition on spin, favored also by Pirani. Another interesting new observation made by Pirani is that Eq. (4.2) implies that the Pauli–Lubański vector (4.10) undergoes Fermi propagation. In Sect. 5, there is an analysis, in terms of components with respect to tetrads, of the discontinuities of the Riemann tensor that occur across the surface of a world-tube of matter.

Note by the Golden Oldie Editor: The reprint of Synge's paper is an exact photocopy of the original text. Pirani's paper was retyped into LaTex in order that a few errors spotted in the original text could be corrected. All the corrections are marked by editorial footnotes, except for quite obvious spelling errors in words that were corrected without marking.

Felix Pirani: a brief autobiography

By Felix Pirani

I was born in London, England on 2 February 1928. I was awarded a B.Sc. at the University of Western Ontario in 1948, M.A. at the University of Toronto in 1949, D.Sc. at the Carnegie Institute of Technology, Pittsburgh, where my supervisor was Alfred Schild, in 1951, and Ph.D. at Cambridge University, where my supervisor was Hermann Bondi, in 1956. I taught at Kings College, London from 1955 until 1983, retiring as Professor of Rational Mechanics. My main research interest was in general relativity, particularly in the theory of gravitational radiation. I collaborated with A.W. Foster, Alfred Schild, Ray Skinner, Ludvik Bass, Hermann Bondi, Ivor Robinson, Gareth Williams, Clive Kilmister, Stanley Deser, Michael Crampin, Jürgen Ehlers, David Robinson, Bill Shadwick, M. Leo, R. A. Leo, L. Martina, and G. Soliani. I developed a strong dislike for black holes and hoped that they would lose plausibility, but I was disappointed. I also worked on solitons and Bäcklund transformations, and on classical mechanics. Michael Crampin and I published a book on applicable differential geometry. I wrote popular articles against nuclear weapons. After I retired I wrote several children's books, one of which was condemned by a British Parliamentary motion for inciting children to alcoholism and violence. My book for young people in favor of nuclear power has recently been translated, without my permission, into Farsi. My main pursuit for the last decade has been making marble mosaics, including several of Penrose–Escher designs; I sometimes give mosaics to people in exchange for donations to Médecins Sans Frontières.

Note by the Golden Oldie Editor: The biography of John Lighton Synge appeared together with another Golden Oldie, in *Gen. Relativ. Gravit.* **34**, 2174 (2002), article DOI 10.1023/A:1021199821507.

Comments to Pirani's biography

By Andrzej Trautman

The above biography of Felix Pirani, written by himself, reflects well his modesty and tendency to self-effacement. Felix played a significant role in the growth of interest and research in general relativity that started in Europe and the USA in the middle of the twentieth century, and himself made several important contributions to the field.

Felix obtained his first doctorate in 1951, working in Pittsburgh with Alfred Schild on the Hamiltonian formalism of the full Einstein theory, with a view to preparing it for covariant quantization; their first paper on the subject [28] appeared at about the same time as those of Dirac and of the Bergmann school.

In 1954 Felix went to the Institute for Advanced Studies in Dublin. There was considerable collaboration and mutual influence between Synge and Pirani there. Synge was then writing his book on *Relativity: The General Theory*. In the words of Synge (see pp. X and XI in [29]), "Dr. Pirani introduced me to the transport law of Fermi which plays an important part in the book, and my attempt to turn Riemannian geometry into observational physics (measure the Riemann tensor!) originated largely in discussions with him…" Following a remark by Ivor Robinson, Felix also drew Synge's attention to an error in his earlier book [30]. Contrary to what is asserted on p. 94 there, not all proper Lorentz transformations are 4-screws: null Lorentz transformations are not. Felix's paper that is reprinted here was written during his stay in Dublin.

Towards the end of 1955, Felix assumed a position at the Department of Mathematics, King's College, London. Hermann Bondi, Clive Kilmister and Felix Pirani formed there the nucleus of a group that, for some time, exerted a major influence on research in the entire field of general relativity. Thanks to their hospitality, King's became the meeting place for many of the scientists most active in the field. In 1957 Felix published the influential paper [31], his first on gravitational radiation. He described there, making it for the first time accessible to the English-speaking community, the original Petrov classification of conformal curvature tensors and suggested its use for an invariant characterization of gravitational radiation. The paper started a flurry of activity on Petrov's classification and its physical significance. The most important early advance was the refinement of the classification by Roger Penrose and the proof of the peeling property of curvature by Ray Sachs and others; much of that work was done at King's.

In 1957 I was privileged to meet Felix in Warsaw, where he came on the invitation of Leopold Infeld. At that time, I had already made a small contribution to Felix's line of thought, by showing that, in agreement with his expectations, the leading part of curvature due to an isolated radiating system is of Petrov type null. Felix invited me to King's; in the spring of 1958 I gave a series of lectures there [32]. That stay and the subsequent visits in the sixties played a decisive role in my scientific life; this is where I met the people who influenced me most: Peter G. Bergmann, Hermann Bondi, Michel Cahen, Josh N. Goldberg, Peter Higgs, Ted Newman, Roger Penrose, Rainer K. Sachs, Alfred Schild, Dennis W. Sciama and Ivor Robinson.

In 1959 there appeared the paper by Bondi et al. [33] containing a new characterization of gravitational plane waves in terms of their symmetries. The authors clearly showed that there are such waves devoid of any real singularities, thus completely resolving the controversy on the nature of the singularities found by Einstein and Rosen in their work of 1936 on exact plane waves; see the historical review [34]. In 1962 Felix gave a summary of the theoretical knowledge of gravitational radiation at the time [35].

The lectures by Felix at Brandeis University [36] included a lucid exposition of rigid motions in relativity (pp. 201–227) and chapters on gravitational radiation, spinors and their applications in general relativity. The two-volume treatise by Penrose and Rindler on *Spinors and space–time*, with its wealth of new ideas and results, is now the standard text on the subject, but beginners still do well by starting with the Brandeis lectures by Pirani.

The volume of papers in honor of Synge [37] contains a fundamental article by Jürgen Ehlers, Felix Pirani and Alfred Schild on the construction of Riemannian geometry of space–time from the projective geometry of free fall and the conformal structure underlying propagation of light; see also [38–40].

In later years, Felix's interests centered around applications of differential geometry [41], especially on equations admitting soliton solutions [42–44]. He prepared a succession of editions of Bertrand Russell's book on relativity [45] and wrote a popular book on cosmology which has been translated into 11 languages [46].

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