How bats have proved the theory of relativity to be wrong[1]

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Abstract: In this didactic article, the theory of special relativity is derived from simple assumptions, somewhat different from the traditional postulates of relativity and constancy of the velocity of light. The basic assumption is that clocks are synchronized by "universal signals". Bats might have assumed them to be provided by sound, but they would have found that elementary clocks do not run in agreement with such a synchronization mechanism; this is described in a fable at the end of the article.

I. INTRODUCTION

Einstein's special relativity theory (SRT) has been accepted and used by physicists for a long time—almost a century. But even now this theory is met with doubts and mistrust by those that are exposed to it for the first time and wish to understand its role and place in the development of science. Sometimes the doubts come from reading popular books on the subject, whose authors present SRT as full of paradoxes. Often, these authors try to impress the reader by simplified, but easy to remember, statements about the contraction of lengths and slowing down of moving clocks. Physicists are also sometimes at fault: having fully absorbed and mastered the theory of relativity, most of them do not feel the need to present its foundation in a careful manner, accessible to the layman.

Traditionally, most expositions of SRT begin with an account of the experiments by Michelson and Morley. These physicists, at the end of the 19th century, by using interferometers constructed for the purpose, have performed measurements intended to determine the motion of the Earth relative to the ether, a hypothetical carrier of the electromagnetic phenomena. The measurements did not detect such motion and have led to the formulation of the *postulate of the constancy of light velocity.* Another general postulate is that of *relativity*, asserting the equivalence of all inertial frames for the description of physical phenomena, including electromagnetism. From those two one derives the form of Lorentz transformations and their physical consequences on the measurement of lengths and time intervals. Such an approach is historically not entirely justified: Einstein, in formulating his theory of 1905, did not even refer to the Michelson and Morley experiments, even though he probably knew about them at the time[2]. Moreover, the negative results of those experiments can be explained on the basis of the *emission hypothesis* of Ritz, according to which light propagates in vacuum with the same velocity with respect to the reference frame of the source[3].

The special theory of relativity is correct in the sense that it describes well—in any case much better than the physics of Galileo and Newton—space-time relations, including those at large velocities among the bodies in motion. This is confirmed by a very impressive wealth of observations of phenomena accompanying motions, accelerations, collisions and decays of elementary particles, nuclei and atoms. Quantum physics sheds a new light on SRT. In particular, the indistinguishability of elementary particles ("all electrons are precisely the same") guarantees the existence of "universal" units of time and length. For this reason, it seems appropriate to begin a lecture on SRT with simple observations ("principles", "postulates") that reflect the present state of physics, rather than by making appeal to the Michelson and Morley experiments of 100 years ago. The purpose of this article is to formulate such a set of postulates and sketch how one can derive from it the elementary predictions of SRT. This approach is very close to that of Hermann Bondi and his k-calculus[4]. In it, an essential role is played by distinguished signals used to synchronize clocks. In SRT they are electromagnetic signals, but, as emphasized already by Einstein[5], one could, in principle, use other "universal" signals. He wrote:

> The theory of relativity is often criticized for giving, without justification, a central theoretical role to the propagation of light, in that it founds the concept of time upon the law of propagation of light. The situation, however, is somewhat as follows. In order to give physical significance to the concept of time, processes of some kind are required which enable relations to be established between different places. It is immaterial what kind of processes one chooses for such a definition of time. It is advantageous, however, for the theory, to choose only those processes concerning which we know something certain. This holds for the propagation of light *in vacuo* in a higher degree than for any other process which could be considered, thanks to the investigations of Maxwell and H. A. Lorentz.

To clarify this point of view and, at the same time, emphasize the truly distinguished place of electromagnetic phenomena in nature, this article concludes with a fable about a civilization of bats. Had they developed physics, they might have used ultrasound to synchronize clocks and define space and time relations. At some point of their history, they would find that elementary phenomena—even their biological clocks—do not run according to the laws of relativity based on sound.

II. THE POSTULATES

At the basis of all classical physics, *with gravitation neglected*, is

Newton's First Law

saying that *there are clocks and reference systems with respect to which free motions occur without acceleration.* In other words, there are coordinates (x, y, z, t) such that free motions of particles are given by *linear* relations among the coordinates.

Throughout the article I consider, for simplicity, only one-dimensional phenomena, i.e. a two-dimensional space-time. To specify an *event* two coordinates, say x and t, suffice. According to the First Law, they are determined up to *linear* transformations,

$$x' = ax + bt + x_0, \quad t' = cx + et + t_0, \tag{1}$$

where a, b, c, e, x_0 and t_0 are constants (real numbers) and $ae - bc \neq 0$. Putting $a = 1, c = 0$ and $e = 1$ in (1) one obtains the well-known *Galilean transformation.* The coefficients a and e are associated with the possibility of changing units and $c \neq 0$ appears, e.g., when on a small portion of the equator one considers a continuously changing solar time.

The First Law, applied to one-dimensional motions, implies that the set of all events is an *affine plane*, i.e. a plane where the Thales theorem holds, but there is no Pythagoras theorem yet, because metric relations are not specified. It is convenient to represent graphically, on that plane, histories of material points and observers: the sets of their events form *world lines* that are straight for free motions and inertial observers. Parallel straight lines represent particles or observers that are at rest one relative to the other. Such observers can agree upon a common unit of time: this is achieved by sending any signals with parallel world lines (Fig. 1). Observers that share in their history a common event—so that their world lines intersect— can take it for beginning of time reckoning, but the First Law, in this case, is not enough to determine a common unit of time (Fig. 2).

Postulate On Universal Signals

To determine a common unit of time for observers in relative motion and to define a coherent method of measuring distances and time intervals, it is necessary to have a *family of universal signals.* On the affine plane it is represented by two sets of parallel straight lines with the property that every point (event) is the intersection of two lines (Fig. 3). There are two distinct families if the velocity of the signals is finite. In Newtonian physics, one accepts the existence of signals that propagate instantaneously—with infinite velocity. In this case, the two families degenerate into one: its lines correspond to spaces of constant *absolute time*.

This postulate replaces the hypothesis of the constancy of the velocity of light. It asserts that the propagation of signals does not depend on the motion of their sources. The parallel nature of the lines of one family means that a photon will never catch up with another photon moving in the same direction. In this article, from now on, thin lines in figures represent the

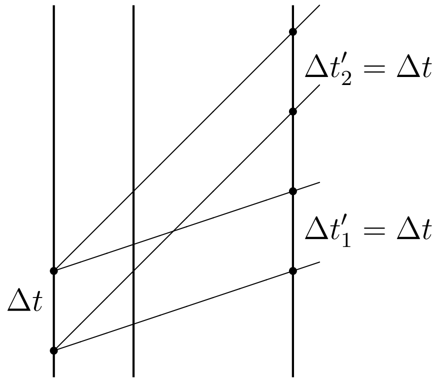

Fig. 1
Observers that are at rest with respect to each other can agree upon a unit of time, but not on its origin.

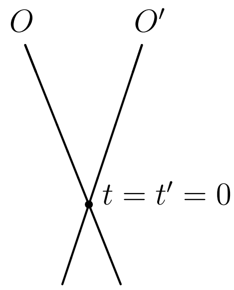

Fig. 2
Observers that share an event can take it for the origin of time.

world lines of the distinguished signals.

Having agreed upon the choice of signals used for the synchronization of clocks and verified that the they satisfy the requirements of the postulate on universal signals, the inertial observers may now accept

A Convention

Concerning The Unit Of Time

As mentioned in Section II, inertial observers that are at rest relative to each other, can agree upon a unit of time using any signals whatsoever. Consider now inertial observers O and O' moving one with respect to the other: their world lines intersect at an event that can be taken for the origin of the times measured by the observers' clocks, $t = t' = 0$ (Fig. 4). Assume now that when the clocks of both observers show 1, they send, one to the other, signals of the preferred, universal type, and note the times α and α' shown on their respective clocks when the signals reach them.

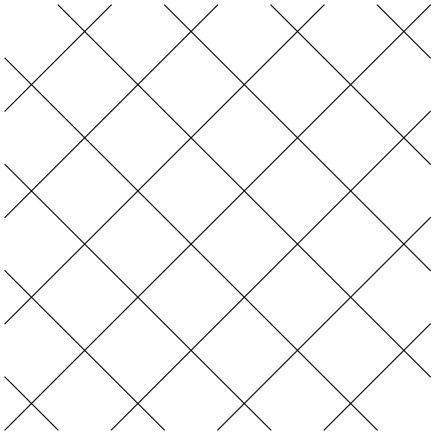

Fig. 3
Two families of parallel lines
represent the world lines of signals.

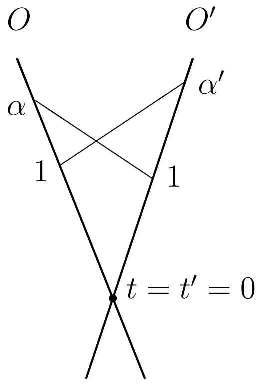

Fig. 4
The convention $\alpha = \alpha'$
expresses the equivalence
of inertial observers.

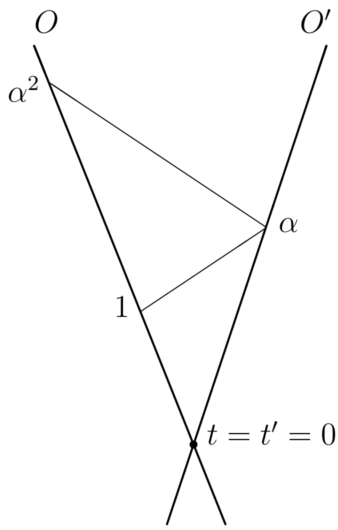

Fig. 5
A consequence of the convention
and of the Thales theorem.

The observer O, assumed to be a gentleman, adjusts his clock to that of the lady O' so that $\alpha = \alpha'$. The coefficient α has a simple physical interpretation: if the signal sent by O is monochromatic and has period T, then the signal received by O' will have period αT. In other words, α is the *Doppler coefficient*. After the adjustment of clock rates, described above, by virtue of the Thales theorem, a signal sent by O at time t, received by O' at time αt and instantaneously sent back, returns to O at time $\alpha^2 t$ shown on the O's clock, Fig. 5. The convention is correct in the sense that it is symmetric—the observers O and O' are on the same footing—and consistent, i.e. transitive with respect to observers: if two pairs of observers, O_1, O_2 and O_2, O_3 adjust, pairwise, their clocks, then the clocks of the observers O_1 and O_3 will run consistently, as can be verified by using the Thales theorem in the situation represented in Fig. 6.

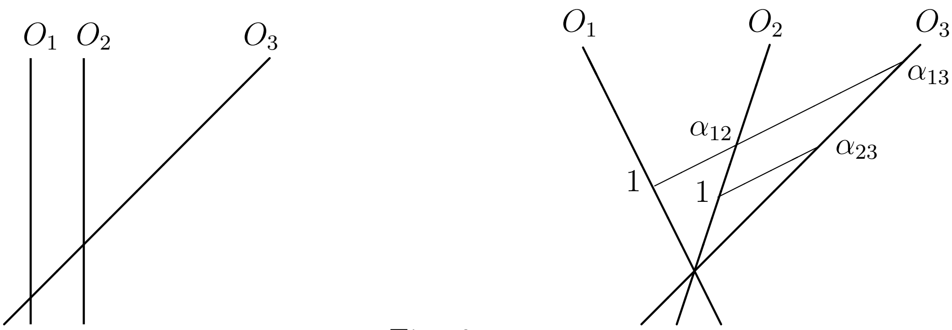

Fig. 6
The convention on the units of time, used by different observers is correct because it is consistent (transitive)
The composition law of the Doppler coefficients is: $\alpha_{13} = \alpha_{12}\alpha_{23}$.

The clocks satisfying our Convention are referred here to as *good*. One should recognize that the Convention does not say anything about the construction of good clocks. For example, if we established radio communication with a distant, extraterrestrial civilization, but could not exchange with it rectilinear light signals, then we would not be able, on the basis of the Convention alone, agree upon what is meant by 1 second. Forgetting this difficulty for a moment, consider now the following

Prescription
for Measurements of Distances and Times

An observer O considers the event B as simultaneous with the event A occurring at time $t = \frac{1}{2}(t_1 + t_2)$, where t_1 and t_2 are, respectively, the moments of time, shown by his (good!) clock, corresponding to the emission and reception of the signals that meet the event B, see Fig. 7. The distance of B from O is proportional to $\frac{1}{2}(t_2 - t_1)$; the coefficient of proportionality c is chosen according to the range of phenomena under consideration. For example, in astronomy, one often chooses $c = 1$ and uses one year as a unit of both time intervals and distances.

Having now a method to measure distances and time intervals, one can define the *relative velocity* of two observers or material points.

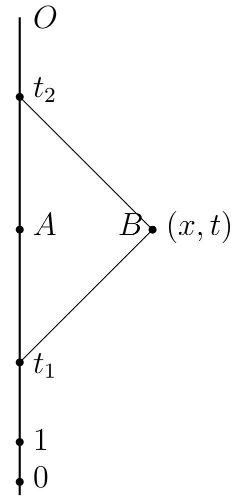

Fig. 7
Observer O considers the events A i B to be simultaneous and ascribes to B the distance $x = \frac{1}{2}c(t_2 - t_1)$ and time $t = \frac{1}{2}(t_1 + t_2)$.

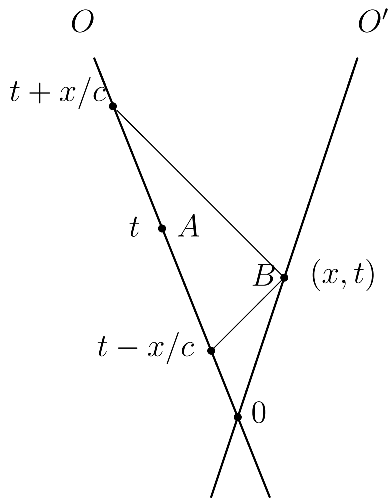

Fig. 8
Interpretation of the coefficient α: from $x/t = V$ and $t + x/c = \alpha^2(t - x/c)$ one obtains formula (2).

The observer O, having ascertained that O' at time t is is at a distance x from him, will declare the velocity of O' to be $V = x/t$ (Fig. 8). On the other hand (Fig. 5), we have $t + x/c = \alpha^2(t - x/c)$ so that

$$\alpha = \sqrt{(1+\beta)(1-\beta)}, \tag{2}$$

where $\beta = V/c$. The observer O considers the events A and B to be simultaneous (cf. Fig. 8 and 11), but the clock of observer O' at B shows $t' = \alpha(t - x/c)$; by virtue of $x = Vt$ and Eq. (2) one obtains the formula for the *relativistic dilation of time*

$$t' = \sqrt{1-\beta^2}\, t. \tag{3}$$

Lorentz transformations are equally easy to derive (Fig. 9): assume that observers O and O' associate with the event Z the coordinates (x, t) and (x', t'), respectively. On the basis of the Convention and the Thales theorem, one has

$$t' - x'/c = \alpha(t - x/c), \quad t + x/c = \alpha(t' + x'/c).$$

This leads at once to the *invariance of the space-time interval*

$$c^2 t'^2 - x'^2 = c^2 t^2 - x^2$$

and the Lorentz formulas

$$x' = (x - Vt)/\sqrt{1-\beta^2}, \quad t' = (t - Vx/c^2)/\sqrt{1-\beta^2}. \tag{4}$$

The so-called twin "paradox" is illustrated on Fig. 10: one of the twins, say the girl O', begins a journey, receding from her brother O with a velocity V very near to that of light, so that $\beta \lessapprox 1$. After some time, she fires the reverse engines of her rocket so as to achieve a velocity opposite to that during the first leg of the journey. Assume that the period of time τ, when O' is undergoing acceleration is much smaller than the time T' of her journey, but large enough to make the acceleration, which is of the order of V/τ, so small that it does not affect her health and the rate of her clock. When the twins meet, they compare their appearance and the times shown by their clocks. The traveller appears to be younger: the relation between the times T and T' shown by the clocks of the twins, when they meet, can be read off from Eq. (3), i.e. $T' \approx \sqrt{1-\beta^2}\, T \ll T$.

In some discussions of the history of those twins, the following aspect—allegedly paradoxical—is emphasized. The relativistic time dilation, given by formula (3), is invariant with respect to the replacement of V by $-V$. Therefore, some people say, *each* of the twins should, throughout the entire journey, see the other one aging at a lower rate than him or herself. To explain in detail the fallacy of this argument, consider, for simplicity, the case of $\tau = 0$. One can now easily determine the rate of the clock of one twin, *measured* by the other, as a function f of the time shown on the clock of the other. For the stationary twin O, the function follows simply from (3): $f(t) = t\sqrt{1-\beta^2}$. In the history of the traveller O', it is necessary to distinguish, from the point of view under consideration, three periods. At first, well before the turning point when the speed of O' changes sign, a light signal sent by O' towards O and sent back with the information about the reading of O's clock, reaches O' when she is still receding from her brother. During that period Eq. (3) applies, $f'(t') = t'\sqrt{1-\beta^2}$. The signal sent by O' at her time t_1 such that $\alpha^2 t_1 = \frac{1}{2}\sqrt{1-\beta^2}\,T$,returns to her at the turning point. For every t'_1 such that $t_1 < t'_1 < \frac{1}{2}\sqrt{1-\beta^2}\,T$ the signal sent by O' towards O reaches O at his time $\alpha t'_1$ and returns to O' at her time $\alpha^2 t'_1$. She considers ???

An Experimental Fact:
Elementary Clocks Are Good

The theoretical considerations outlined above are relevant to the real world because *there are many good clocks*, satisfying the requirement of the Convention without the need for actual adjusting of their rates. We owe the knowledge of such clocks to the physics of the micro world

[1] This is an expanded version of the author's article in Polish: "O tym, jak nietoperze obaliły teorię względności", Postępy Fizyki , – ().

[2] See pp. 115–119 in A. Pais, *'Subtle is the Lord...'* The Science and the Life of Albert Einstein (Oxford University Press, Oxford, 1982).

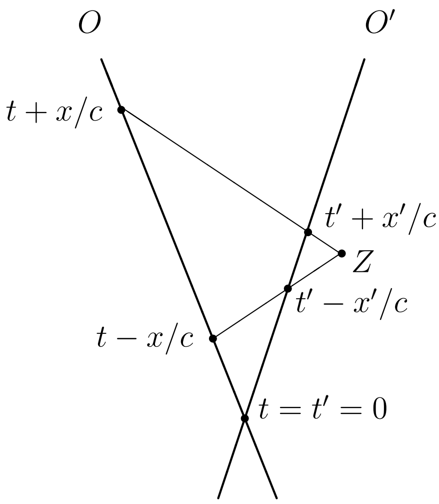

Fig. 9
Lorentz transformation.
Observers O and O' associate with Z, coordinates (x, t) and (x', t'), respectively.

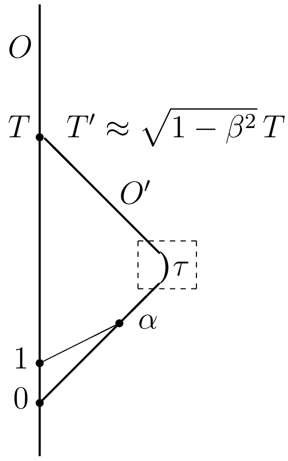

Fig. 10
"The twin paradox"

[3] See, e.g. pp.5–8 in W. Pauli, *Theory of Relativity* (Pergamon Press, London, 1958).

[4] H. Bondi, *Relativity and common sense* (Doubleday, Garden City,1964); see also W. Kopczyński and A. Trautman, *Spacetime and Gravitation* (PWN and Wiley, Warszawa and Chichester, 1992).

[5] A. Einstein, *The meaning of relativity*, Fifth edition (Princeton University Press, Princeton, 1956), pp. 28–29.