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Spin structures on real projective quadrics

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Projective quadrics are known to be conformal compactifications of Euclidean spaces. In particular, the (projective) real quadric $Q_{p,q} = (S_p \times S_q)/\mathbb{Z}_2$ is associated, in this manner, with the flat space $\mathbb{R}^{p+q}$ endowed with a metric tensor of signature (p, q). For p and q positive, the quadric $Q_{p,q}$ is orientable iff $p + q$ is even. The quadric has two natural metrics, invariant with respect to the action of $O(p + 1) \times O(q + 1)$: a proper Riemannian one and a pseudo-Riemannian metric of signature (p, q). This paper contains an explicit description of spin structures on real, even-dimensional quadrics for both metrics, whenever these structures exist. In particular, it is shown that, for p and q even positive, the proper (pseudo-Riemannian) metric gives rise to two inequivalent spin structures iff $p + q \equiv 2 \pmod 4$ $(p + q \equiv 0 \pmod 4)$. If p and q are odd and > 1, then there is no spin structure for either metric whenever $p + q \equiv 0 \pmod 4$; otherwise, there are two spin structures for each of the metrics. There always exist spin structures on real quadrics with a Lorentzian metric, i.e., when p and q are odd and p or $q = 1$.

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Dedicated to Roger Penrose

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