

COMMENTS ON THE PAPER BY ELIE CARTAN: SUR UNE GENERALISATION DE
LA NOTION DE COURBURE DE RIEMANN ET LES ESPACES A TORSION

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This paper is a real gem, written in a style characteristic of Elie Cartan: it contains important new ideas, but no precise definitions, theorems or equations.

The author generalizes the notion of parallel transport of vectors, introduced by Levi Civita. This generalization is motivated by physical considerations: Cartan refers to his earlier paper on the stress-energy tensor in Einstein's theory and to the work of the brothers E. and F. Cosserat on continuous media with an intrinsic angular momentum.

The geometry considered by Cartan is that of a three-dimensional manifold with a metric tensor g and a linear connection ω which is Euclidean - or metric - i.e. compatible with g . The condition of compatibility may be written as

$$Dg_{\mu\nu} = 0, \quad (1)$$

where D is the exterior covariant derivative, $Dg_{\mu\nu} = dg_{\mu\nu} - \omega_{\mu}^{\rho} g_{\rho\nu} - \omega_{\nu}^{\rho} g_{\rho\mu}$, and $\mu, \nu, \rho = 1, 2, 3$.

Cartan emphasizes that eq.(1) does not completely define the connection form ω_{ν}^{μ} , namely

$$\omega_{\nu}^{\mu} = \gamma_{\nu}^{\mu} + \kappa_{\nu}^{\mu}, \quad (2)$$

where γ is the Levi Civita connection and the tensor-valued form $\kappa_{\mu\nu} = g_{\mu\rho} \kappa_{\nu}^{\rho}$ is skew in the pair (μ, ν) , but otherwise arbitrary. He interprets formula (2) by saying that infinitesimal parallel

