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9

Fiber Bundles, Gauge Fields, and Gravitation

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1. Introduction and Motivation

1.1. Physicists were using concepts that are now part of the theory of fiber bundles before mathematicians introduced the notion of a bundle. For example, the phase space of classical mechanics and statistical physics coincides with the cotangent bundle of a configuration space. The derivation by Dirac of the formula for the strengths of magnetic poles is equivalent to the classification of circle bundles over S_2 by their Chern numbers.[(1)] In this respect, the situation of physicists can be likened to that of Monsieur Jourdain *qui fait de la prose sans le savoir.* There thus arises the question whether it is worth while to learn the language and use the methods of fiber bundles since so far it has been possible to do without them. It is hard to give a straightforward and convincing answer to this question; probably the only reasonable thing to say is "the future will tell." My personal opinion is that at least some concepts of fiber bundle theory will become an established part of mathematical physics because fiber bundles provide a natural and convenient framework for discussing the concepts of relativity and invariance, describing gravitation and other gauge fields, defining the notion of induced representations, and giving a geometrical interpretation to quantization and the canonical formalism of particles and fields. Fiber bundles provide a convenient language for dealing with local problems of differential geometry and field theory. They are necessary to understand and solve global, topological problems, such as those arising in connection with magnetic poles and instantons.

For a long time, Einstein searched for a "unified" geometrical theory of gravitational and electromagnetic forces. The success of his attempts, based

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