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LETTER TO THE EDITOR

A class of null solutions to Yang–Mills equations

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Abstract. A class of null solutions of Maxwell's equations is generalised to Yang–Mills theory. The plane-fronted, non-Abelian waves, recently found by Coleman, are among the null solutions.

Consider a plane electromagnetic wave, propagating in the z direction in Minkowski space. Its electric and magnetic fields are

$$\boldsymbol{E} = a\boldsymbol{e}_x + b\boldsymbol{e}_y, \qquad \boldsymbol{B} = a\boldsymbol{e}_y - b\boldsymbol{e}_x, \tag{1}$$

where a and b are arbitrary functions of $u = t - z$, $\boldsymbol{e}_x = \text{grad } x$ and $\boldsymbol{e}_y = \text{grad } y$. This field may be derived from a potential (A_μ), $\mu = 1, 2, 3, 4$, which will be represented by the 1-form $A = A_\mu \, \mathrm{d}x^\mu$. A possible choice is

$$A = (ax + by + c) \, \mathrm{d}u \tag{2}$$

where c is another function of u. Indeed, the electromagnetic field is given by the 2-form

$$F = (a \, \mathrm{d}x + b \, \mathrm{d}y) \wedge \mathrm{d}u. \tag{3}$$

Clearly, the function c occurring in (2) may be eliminated by a gauge transformation without affecting either a or b.

The potential (2) is suitable for a generalisation to non-Abelian gauge fields, including gravitation, and to a class of null 'spherical' waves.

Consider the line element

$$2 \, \mathrm{d}u(\mathrm{d}v + H \, \mathrm{d}u) - P^2(\mathrm{d}x^2 + \mathrm{d}y^2) \tag{4}$$

on a four-dimensional manifold M, referred to coordinates x, y, u and v. It includes Minkowski space in various ways, i.e. in different coordinate systems. In particular, if

$$P = 1 \qquad H = 0 \qquad u = t - z \qquad v = \tfrac{1}{2}(t + z) \tag{5a}$$

then the line element reduces to

$$\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2. \tag{5b}$$

Similarly, if

$$P = v[1 + \tfrac{1}{4}(x^2 + y^2)]^{-1} \qquad H = \tfrac{1}{2} \qquad u = t - r \qquad v = r \tag{6a}$$
$$x + \mathrm{i}y = 2e^{\mathrm{i}\varphi} \cot \tfrac{1}{2}\vartheta$$

then the line element is

$$\mathrm{d}r^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\vartheta^2 + \sin^2 \vartheta \, \mathrm{d}\varphi^2). \tag{6b}$$

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