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Spin and Torsion May Avert Gravitational Singularities

IT has long been recognized that isotropic models of the world are singular. The models, be they Newtonian or based on Einstein's equations of general relativity, lead to the prediction of at least one moment in the history of the Universe with an infinite density of matter. A similar singularity has been shown to occur as a result of spherically symmetric gravitational collapse of a cloud of dust[1]. For some time it was thought that the singularities might be a consequence of the high symmetry of the models.

Penrose[2], Geroch[3], and Hawking[4] have shown that the occurrence of such singularities is a general prediction of Einstein's theory of gravitation; it has nothing to do with the symmetry of the models, and is independent of the detailed form of the stress tensor, provided that the latter satisfies a reasonable "energy condition". It seems that in Einstein's theory it is possible to obtain regular models of the Universe only at the price of introducing into the equations a positive λ term, representing "cosmic repulsion", whereas local collapse leads inescapably to singularities[5]. This is also true in some of the modifications of the theory of general relativity, such as the one proposed by Brans and Dicke[6].

About a year ago, I conjectured[7] that the singularities of gravitational collapse and cosmology might be prevented by the direct influence of spin on the geometry of space-time, which is taken into account in the Einstein–Cartan theory of gravitation. This is a theory originated by Cartan[8,9], independently formulated by Sciama[10,11] and Kibble[12], and developed by Hehl[13] and myself[14]. According to Cartan, the geometry of space-time is determined by a metric tensor and a linear connexion which are compatible with each other: the scalar product of any two vectors is unchanged by parallel transport of these vectors. By contrast with Einstein's theory, the tensor

$Q^i{}_{jk}$ of torsion is not required to vanish, but is related to the tensor $s^i{}_{jk}$ describing the density of intrinsic angular momentum. The field equations are[7,12]

$$R_{ij}-\tfrac{1}{2}g_{ij}g^{kl}R_{kl}=8\pi Gc^{-4}t_{ij} \qquad (1)$$

$$Q^i{}_{jk}-\delta^i{}_jQ^l{}_{lk}-\delta^i{}_kQ^l{}_{jl}=8\pi Gc^{-4}s^i{}_{jk} \qquad (2)$$

where g_{ij} is the metric tensor, $R_{ij}=g^{kl}\ R_{ikjl}$, R_{ijkl} is the curvature tensor formed from the connexion, t_{ij} is the (asymmetric) energy–momentum tensor, G and c are the gravitational constant and the velocity of light, respectively, and the indices i, j, k, l, run from 0 to 3. The system (1) and (2) may be derived from a principle of least action; it reduces to the system of Einstein's equations when the density of intrinsic angular momentum is neglected.

Recently, Kopczyński[15] constructed a class of non-singular cosmological models based on the Einstein–Cartan theory of gravitation. The models provide a lower bound for the minimum radius of the world, corresponding to the hypothetical "bounce" and presumably occurring during the hot stage of the development of the Universe. One of the simplest models is for a Universe filled with a spinning dust characterized by its four-dimensional velocity u^i, density of mass ρ, and density of spin $s^i{}_{jk}=u^i\ S_{jk}$, $u^k\ S_{jk}=0$. These assumptions are compatible with a Robertson–Walker line element ds,

$$ds^2=c^2dt^2-R(t)^2(dx^2+dy^2+dz^2)$$

and $u^i=\delta_0{}^i$, with $x^0=ct$, $x^1=x$, $x^2=y$, and $x^3=z$. Assuming that the spins are aligned along the x axis, $S_{23}=\sigma$ and $S_{jk}=0$ for $j+k\neq 5$, and making use of equation (2) to determine the torsion, one can reduce equation (1) to a modified Friedmann equation

$$\frac{1}{2}\dot{R}^2-\frac{GM}{R}+\frac{3G^2S^2}{2c^4R^4}=0 \qquad (3)$$

supplemented by the conservation laws of mass and spin,

$$M=\frac{4}{3}\pi\rho R^3=\text{constant and } S=\frac{4}{3}\pi\sigma R^3=\text{constant}$$

The last term on the left side of equation (3) plays the role of a "repulsive potential", which is effective at small values of R and prevents the solution from ever approaching zero. In fact, the equation may be solved exactly.

$$R(t)=\left(\frac{3GS^2}{2mc^4}+\frac{9GMt^2}{2}\right)^{1/3}$$

At $t=0$, the radius of a sphere containing N particles of mass $m=M/N$ and spin $\frac{1}{2}\ \hbar=S/N$ is

$$R_{\min}=(3NG\hbar^2/8mc^4)^{1/3}$$

If m is the mass of a neutron, then

$$R_{\min}\simeq 3\times 10^{-27}N^{1/3}\ \text{cm}$$

For $N=10^{80}$, a figure which is often quoted as representing the total number of baryons in the part of the Universe accessible to observation, $R_{\min}$ is of the order of 1 cm. This may appear to be a rather small size for the Universe, but it is very large when compared with the Planck length $(G\hbar/c^3)^{1/2}\simeq 1.6\times 10^{-33}$ cm, which has been considered as providing the only natural limitation on the validity of classical, Einsteinian cosmology[16–18]. In my model, the density of matter at $t=0$ is of the order of $m^2c^4/G\hbar^2\simeq 10^{55}$ g cm^{-3}. It is much smaller than the density $c^5/G^2\hbar\simeq 10^{94}$ g cm^{-3} at which the quantum effects of the gravitational field are presumed to play a dominant part. At $t=0$, the torsion is of the same order of magnitude as the inverse Compton wavelength of the particles. For $t\gg\tau=\hbar/mc^2\simeq 10^{-23}$ s the model coincides with the corresponding solution of Einstein's equations. Therefore any considerations concerning the hadronic era of the Universe, and its subsequent development, will be little affected by the introduction of torsion in cosmology. According to Kundt[19], "the first 10^{-23} s after the big bang are admittedly beyond present theory".

Is there any reason to believe that spins were correlated during the hottest stage of the evolution of the Universe, say, for $|t|\lesssim\tau$? A mechanism is provided by a cosmic magnetic field H, for, if it exists at all, it may successfully compete with increasing temperature T, provided that the flux is conserved, $HR^2=$constant. As $TR=$constant, the ratio $\mu H/kT$ behaves like $1/R$ and may have been so large in the past that the magnetic field aligned all spins. More precisely, if μ is the nuclear magneton and H_0 is the (unknown) present value of the intergalactic magnetic field, then $\mu H(t)/kT(t)\simeq 10^{20}H_0/R(t)$, where H_0 is in gauss and $R(t)$ in cm.

The model described here suffers from at least two defects: it neglects both the pressure and the magnetic field energy in the description of the hot stage of the development of the Universe. It would be interesting to know whether a closed cosmological model would also bounce because of torsion. An indication that it would is provided by a solution of the Einstein–Cartan equations with a spherically-symmetric distribution of spins, in which the Friedmann singularity is again averted[20]. Torsion arising from the increasing alignment of spins in a collapsing magnetic star may prevent the occurrence of singularities, even after the star has crossed its event horizon.