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GOLDEN OLDIE EDITORIAL

Editorial note to: Myron Mathisson, The mechanics of matter particles in general relativity

and to: New mechanics of material systems

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The two papers by Mathisson that appear here as Golden Oldies [1,2], contain an implicit polemic with Einstein and his approach to the problem of motion. In the first paper [1], Mathisson shows that nonlinearity of the field equations is not essential for obtaining from them the equations of motion. After giving a neat description of the Liénard–Wiechert potential, Mathisson treats the analogous problem for a weak gravitational field $h_{\mu\nu}$. He introduces, following Einstein, the tensor $\psi_\mu^\nu = h_\mu^\nu - \frac{1}{2}\delta_\mu^\nu h_\rho^\rho$ so that the linearized field equations in empty space are equivalent to $\Box\psi_\mu^\nu = 0$ and $\psi_{\mu,\nu}^\nu = 0$. The tensor field $\psi^{\mu\nu} = m^{\mu\nu}(u)/r$—where u is a suitably defined retarded time—satisfies the wave equation and the Einstein condition $\psi_{\mu,\nu}^\nu = 0$ implies $m^{\mu\nu} = m\dot{z}^\mu\dot{z}^\nu$, $\dot{m} = 0$ and the equation of motion $\ddot{z}^\mu = 0$.

A few months after returning from Kazan (see his biography below), Mathisson sent for publication his most important paper [2] where he introduced the notion of a 'gravitational skeleton' and gave a derivation of the coupling between spin and curvature. In Mathisson's definition of the gravitational skeleton one can see the germ of the idea of a distribution, as later introduced by Laurent Schwartz: Mathisson uses 'test functions' $p_{\mu\nu}$ and the equation

The republications of the original Mathisson papers can be found in this issue following the editorial note and online via doi:10.1007/s10714-010-0938-z (The mechanics of matter particles in general relativity) and via doi:10.1007/s10714-010-0939-y (New mechanics of material systems).

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$$\int_D T^{\mu\nu} p_{\mu\nu} d^4x = \int_L \left(m^{\mu\nu} p_{\mu\nu} + m^{\mu\nu\rho} p_{\mu\nu;\rho} + \cdots \right) ds$$

to replace the continuous energy-momentum tensor $T^{\mu\nu}$ filling a world-tube D by an equivalent—as far as the external field is concerned—distribution with support on a time-like world-line $L \subset D$. Taking $p_{\mu\nu} = \xi_{\mu;\nu} + \xi_{\nu;\mu}$ and using the conservation law $T^{\mu\nu}{}_{;\nu} = 0$, Mathisson proves that the above integral over L vanishes for arbitrary ξ vanishing at both ends of the world-line ('Mathisson's variational principle'). He then uses this principle to derive the equations of motion, which in modern form are

$$\begin{aligned} \dot{p}_\mu + \frac{1}{2} R_{\mu\nu\rho\sigma} u^\nu s^{\rho\sigma} &= 0, \\ \dot{s}_{\mu\nu} + u_\mu p_\nu - u_\nu p_\mu &= 0, \end{aligned}$$

where p^μ and $s^{\rho\sigma}$ are, respectively, the momentum and spin (intrinsic angular momentum) of a body moving in space–time with curvature described by the Riemann tensor $R_{\mu\nu\rho\sigma}$; dots denote covariant derivatives in the direction of the 4-velocity u^μ.

In October of 2007, there was held, at the Stefan Banach International Mathematical Center in Warsaw, a conference devoted to Myron Mathisson, his life, work, and influence on current research. The proceedings of the conference were published, as a special issue of *Acta Physica Polonica B: Proceedings Supplement* **1** No.1 (2008) and are freely available at the address http://th-www.if.uj.edu.pl/acta/sup1/t1.htm. In particular, the article by T. Sauer & A. Trautman contains a biography of Mathisson and a list of all his scientific publications. W. G. Dixon gave a report on *Mathisson's New Mechanics: Its Aims and Realisation* and S.R. Czapor & R.G. McLenaghan on *Hadamard's Problem of Diffusion of Waves* and Mathisson's contribution to its solution. These and other papers presented at the conference contain a fairly thorough discussion of Mathisson's work and its subsequent developments. For this reason, the Editors of the Golden Oldies felt that there was no need to present here a detailed analysis of Myron Mathisson's contributions.

Myron Mathisson: a brief biography

By Andrzej Trautman

Myron Mathisson (1897–1940) was a Polish Jew, born and educated in Warsaw, where he obtained, in 1931, the Ph. D. degree under the supervision of Czesław Białobrzeski, a professor of theoretical physics at the University.

In 1929 Mathisson wrote, in French, a letter to Einstein, criticizing, in rather strong words, Einstein's approach to the problem of equations of motion in general relativity and outlining his own ideas on the subject. Einstein answered by inviting Mathisson to Berlin. Mathisson declined the invitation on the ground that he was not yet prepared to collaborate with Einstein; in the following years Einstein and Mathisson exchanged about 20 letters that are accessible at the Albert Einstein Archives at the Hebrew University of Jerusalem. On several occasions, Einstein supported Mathisson to obtain fellowships to pay scientific visits abroad.

In a series of six papers published in the 1930s, Mathisson outlined a new method of deriving equations of motion of bodies in general relativity. He associated with such bodies a distribution with support on a time-like world-line, representing the motion of their center of mass. In particular, he showed that for a body with an intrinsic angular momentum (spin), this world-line is no more a geodesic: Mathisson discovered a new interaction between spin and Riemannian curvature; the resulting equation is now associated with the names of Mathisson and Papapetrou. The latter published in 1951 another derivation of the coupling between curvature and spin. Mathisson's papers contained an early explicit use of the geometry of null (optical) elements to describe fields in relativity. He gave a new derivation of the equation of motion of a radiating electron.

In 1932, Mathisson obtained a habilitation that allowed him to use the title of 'docent' (analog of the German *Privat-Dozent*) and give lectures at the University of Warsaw, but did not imply a permanent, salaried position. In spite of Einstein's support, he never obtained an academic post in Poland.

His visits to Paris (1935 and 1939) and a collaboration with Jacques Hadamard were particularly fruitful. Mathisson developed a new method to analyze the properties of the fundamental solution of linear hyperbolic differential equations and used it to resolve Hadamard's conjecture on the class of equations for which the Huygens principle in four space–time dimensions is valid. Mathisson's work and his paper of 1939 so impressed Hadamard that, after Mathisson's death, he dedicated to him his paper on *The Problem of the Diffusion of Waves*, Ann. Math. **42** (1942) 510–522.

Mathisson spent the academic year 1936–37 lecturing at the University of Kazan. From the fall of 1937 to the spring of 1939 he lived in Cracow and collaborated there with Jan Weyssenhoff, a professor of theoretical physics at the Jagiellonian University.

Mathisson's ideas were further developed, during and after World War II, by Weyssenhoff and his students. In the spring of 1939 Mathisson went to Cambridge and continued there his work on the problem of motion. He impressed the physicists there so that, when in 1940 he died of tuberculosis, Dirac edited and presented for publication the notes left by Mathisson; he also wrote Mathisson's obituary notice that appeared in Nature.

References

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